Solutions for R&D to Design

Considerations about Heat Exchanger Temperature Differences

December 2, 2011

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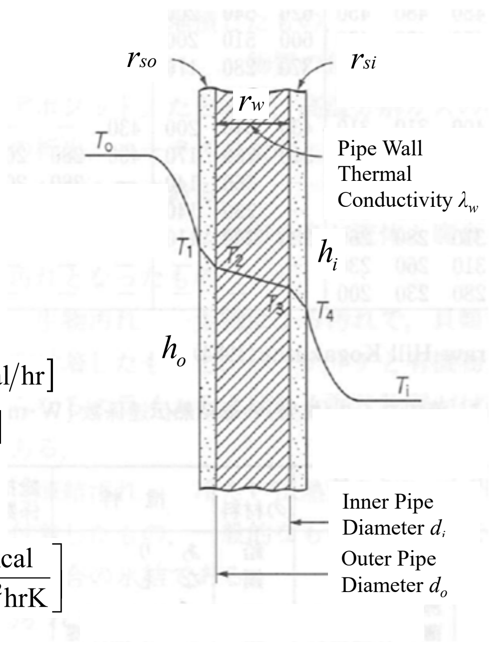
Introduction

- The approximate equation widely used as the heat transfer equation for heat exchangers is "$Q = U*A*\Delta T$".
- The logarithmic mean temperature difference (LMTD) is commonly used as the temperature difference (ΔT) in the heat transfer equation, but there are cases in which it is directly applied to systems with phase changes without consideration on how the LMTD is derived.
- In this study, we will examine the implications of applying the LMTD to systems in which phase changes occur.

Heat Exchanger Heat Transfer Equation

$$Q = U \cdot A \cdot \Delta T$$

$$\frac{1}{U} = \frac{1}{h_o} + r_{so} + r_{wo} + r_{sio} + \frac{1}{h_{io}}$$



Q	Heat Exchange Rate	[kcal/hr]
A	Heat Transfer Area	[m^2]
ΔT	Temperature Difference	[K]
U	Overall Heat Transfer Coefficient	$\left[\frac{\text{kcal}}{\text{m}^2\text{hrK}}\right]$
h_o	Pipe Outer Film Heat Transfer Coefficient	
h_i	Pipe Inner Film Heat Transfer Coefficient	
r_{so}	Pipe Outer Fouling Resistance (Fouling Factor)	$\left[\frac{\text{m}^2\text{hrK}}{\text{kcal}}\right]$
r_{si}	Pipe Inner Fouling Resistance (Fouling Factor)	
r_w	Pipe Wall Resistance	

Derivation of Logarithm Mean Temperature Difference

Temperature Difference between Pipe Wall and Fluid:
When steam (at a constant temperature) is used to heat a fluid flowing in a pipe, the temperature of the fluid varies along the length of the pipe (Fig. 3.6).
In this case, the relationship between the heat exchange rate Q and the temperature difference is expressed by the following equation.

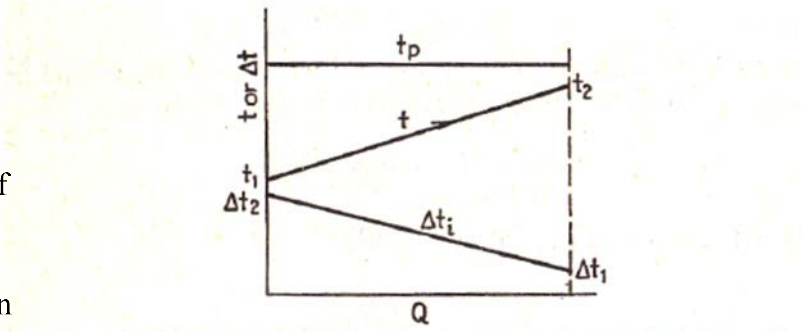


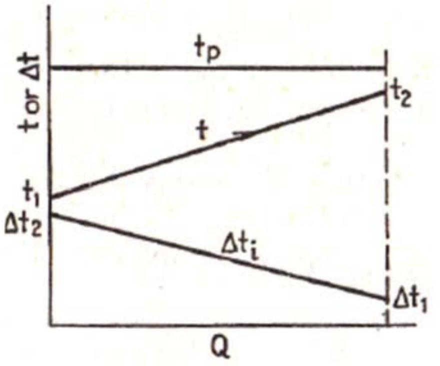
FIG. 3.6. Temperature difference between a fluid and a pipe wall.

$$Q = h_i \cdot A_i \cdot (t_P - t) = h_i \cdot A_i \cdot \Delta t_i \qquad \text{... (1)}$$

Q	Heat Exchange Rate	[kcal/hr]
A_i	Heat Transfer Area	[m^2]
t_P	Pipe Wall Temperature	[K]
Δt_i	Pipe Wall and Fluid Temperature Difference	[K]
h_i	Film Heat Transfer Coefficient	[kcal/m^{2}hrK]

"Processing Heat Transfer",
Kern,McGRAW-HILL

Derivation of Logarithm Mean Temperature Difference



Assuming that the specific heat of the fluid and the heat transfer coefficient are constant, the temperature difference changes linearly in proportion to the heat exchange rate.

$$dQ = h_i \cdot dA_i \cdot \Delta t_i \qquad \ldots (2)$$

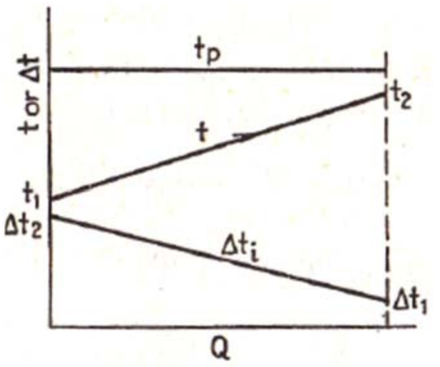
At this time, the gradient of the temperature difference Δt_i is expressed as a function of the heat exchange rate Q.

$$\frac{d\Delta t_i}{dQ} = \frac{\Delta t_2 - \Delta t_1}{Q} \qquad \ldots (3)$$

$$\Delta t_2 = t_P - t_1$$

$$\Delta t_1 = t_P - t_2$$

Derivation of Logarithm Mean Temperature Difference



Substituting dQ in Eq. (3) with Eq. (2) yields:

$$\frac{h_i \cdot dA_i}{Q}\left(\Delta t_2 - \Delta t_1\right) = \frac{d\Delta t_i}{\Delta t_i} \qquad \ldots (4)$$

Integrating Eq. (4) yields:

$$\frac{h_i}{Q}\left(\Delta t_2 - \Delta t_1\right)\int_0^{A_i} dA_i = \int_{\Delta t_1}^{\Delta t_2} \frac{d\Delta t_i}{\Delta t_i}$$

$$\frac{h_i \cdot A_i}{Q}\left(\Delta t_2 - \Delta t_1\right) = \ln\left(\frac{\Delta t_2}{\Delta t_1}\right)$$

$$\therefore Q = h_i \cdot A_i \frac{\left(\Delta t_2 - \Delta t_1\right)}{\ln\left(\Delta t_2 / \Delta t_1\right)} \qquad \ldots (5)$$

The term $\dfrac{\left(\Delta t_2 - \Delta t_1\right)}{\ln\left(\Delta t_2 / \Delta t_1\right)}$ in Eq. (5) is called the Logarithmic Mean Temperature Difference, or LMTD when abbreviated.

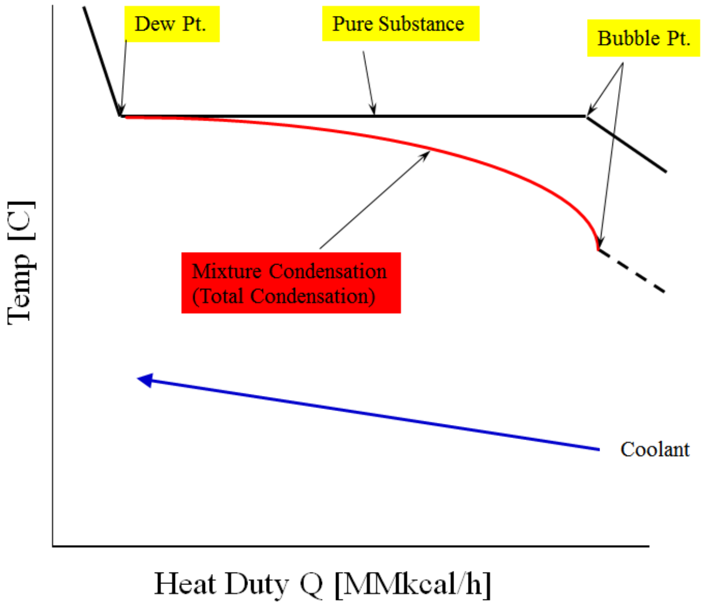
Temperature Change in Condenser *

In a condenser, heat is removed by a refrigerant, and a temperature profile as shown in the right figure is obtained.

In particular, a condenser at the top of a distillation column is often used to subcool the heat exchanger outlet to prevent pump cavitation.

In this type of condenser, we will examine the influence of the temperature difference on the heat transfer area.

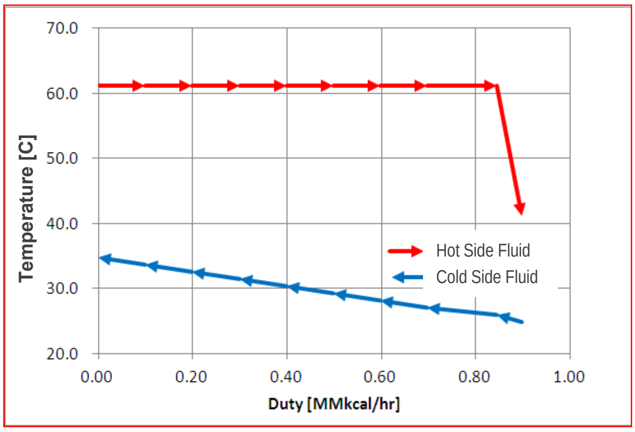
Under Constant Pressure Conditions

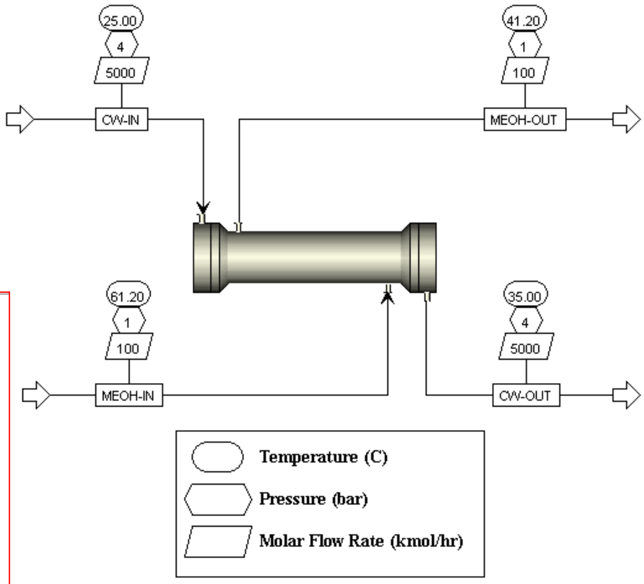


* For the sake of simplicity, it is assumed that a single pass shell & tube (or double tube) heat exchanger is used and that pressure losses can be ignored.

Pure Substance Condensation (Subcool)

Consider an example where methanol in the saturated vapor state is condensed using CW of 25 → 35 [C], and then further cooled to a subcooled degree of 20 [C].





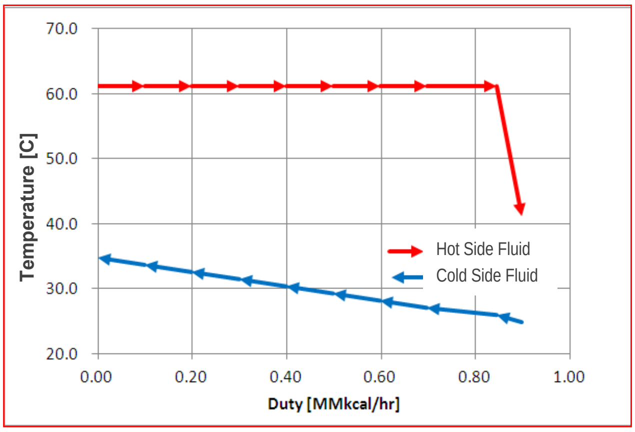
[Study Conditions]

Heat Exchange Rate	: 0.90 [MMkcal/hr]
Methanol	: 61.2[C]→41.2[C]
CW	: 25[C]→35[C]

Calculation of Heat Transfer Area using LMTD

When the logarithmic mean temperature difference is used:

$$LMTD = \frac{(61.2-35)-(41.2-25)}{\ln((61.2-35)/(41.2-25))}$$
$$\fallingdotseq 20.8[C]$$



Assuming the overall heat transfer coefficient (U)= 500 [kcal/m^{2}hrK] the heat transfer area (A) is 86.3 [m^2]

$$A = \frac{Q}{U \cdot LMTD} = \frac{0.90e6}{500 \times 20.8}$$
$$\fallingdotseq 86.3\,[\mathrm{m}^2]$$

Heat Transfer Calculation using Weighted Mean Temperature Difference (WMTD)

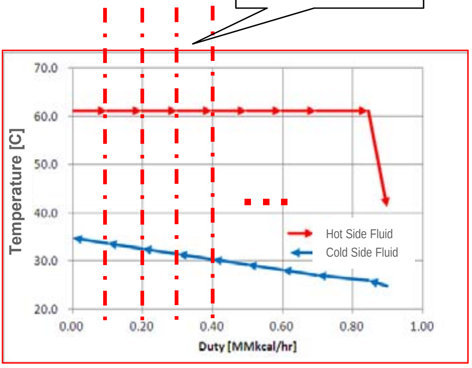
When the weighted mean temperature difference is used:

$$\frac{1}{WMTD} = \frac{1}{\dot{Q}_T}\left[\left(\frac{\dot{Q}}{\Delta T_{LMTD}}\right)_1 + \left(\frac{\dot{Q}}{\Delta T_{LMTD}}\right)_2 + ... + \left(\frac{\dot{Q}}{\Delta T_{LMTD}}\right)_n\right]$$

$$WMTD = 1/(0.0297\ /\ 0.90) = 30.2$$

$$\left(\frac{\dot{Q}}{\Delta T_{LMTD}}\right)_n$$

	Hot Side Fluid	Cold Side Fluid				
Heat Load	Temperature	Temperature	Zone ΔT1	Zone ΔT2	Zone LMTD	q / LMTD
[MMkcal/h]	[C]	[C]				
0.00	61.20	35.00				
0.10	61.20	33.89	26.20	27.31	26.75	0.0037
0.20	61.20	32.78	27.31	28.42	27.87	0.0036
0.30	61.20	31.67	28.42	29.54	28.98	0.0034
0.40	61.20	30.56	29.54	30.65	30.09	0.0033
0.50	61.20	29.44	30.65	31.76	31.20	0.0032
0.60	61.20	28.33	31.76	32.87	32.31	0.0031
0.70	61.20	27.22	32.87	33.98	33.42	0.0030
0.84	61.20	26.11	33.98	35.09	34.53	0.0043
0.90	41.20	25.00	35.09	16.20	24.44	0.0022
					Sum	0.0297



LMTD =	20.80	[m2]
WMTD =	30.19	[m2]

A LMTD =	86.29	[m2]
A WMTD=	59.47	[m2]

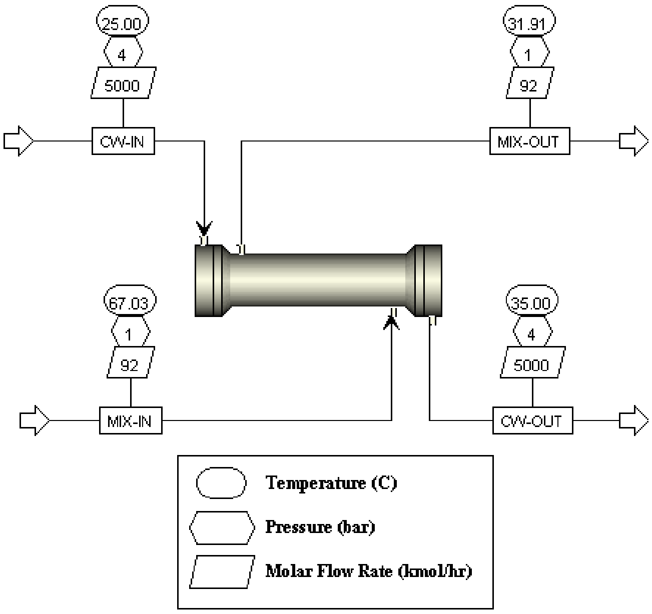
Ratio	45.1%

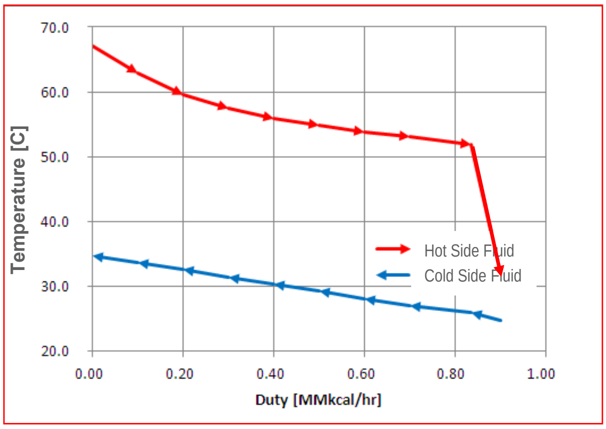
$$A = \frac{Q}{U \cdot WMTD} = \frac{0.90e6}{500 \times 30.2}$$
$$\fallingdotseq 59.5\,[\mathrm{m}^2]$$

The LMTD Base is equivalent to a heat exchanger with a margin of almost 50%

Mixture Condensation (Subcool)

Consider an example in which a mixture in a saturated vapor state is condensed using CW of 25 → 35 [C], and then further cooled to a subcooled degree of 20 [C].





[Study Conditions]

Heat Exchange Rate : 0.90 [MMkcal/hr]

Mixture : 67.0[C]→31.9[C]

CW : 25[C]→35[C]

LMTD vs WMTD (Subcool)

	Hot Side Fluid	Cold Side Fluid				
Heat Load	Temperature	Temperature	Zone ΔT1	Zone ΔT2	Zone LMTD	q / LMTD
[MMkcal/h]	[C]	[C]				
0.00	67.03	35.00				
0.10	62.88	33.89	32.03	28.99	30.49	0.0033
0.20	59.67	32.78	28.99	26.89	27.93	0.0036
0.30	57.48	31.67	26.89	25.82	26.35	0.0038
0.40	55.96	30.56	25.82	25.41	25.61	0.0039
0.50	54.81	29.44	25.41	25.37	25.39	0.0039
0.60	53.87	28.33	25.37	25.53	25.45	0.0039
0.70	53.02	27.22	25.53	25.80	25.67	0.0039
0.83	51.91	26.11	25.80	25.80	25.80	0.0052
0.90	31.91	25.00	25.80	6.91	14.34	0.0045
					Sum	0.0360

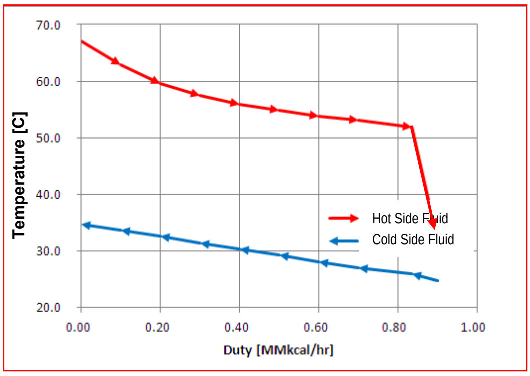
Overall		
ΔT1	ΔT2	LMTD
32.03	6.91	16.38

LMTD = 16.38 [m2]
WMTD = 24.95 [m2]

A LMTD = 109.55 [m2]
A WMTD= 71.94 [m2]

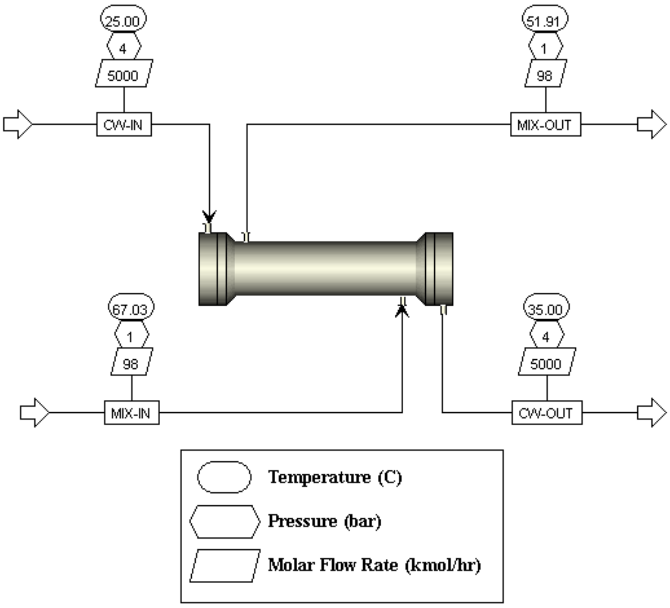
Ratio	52.3%

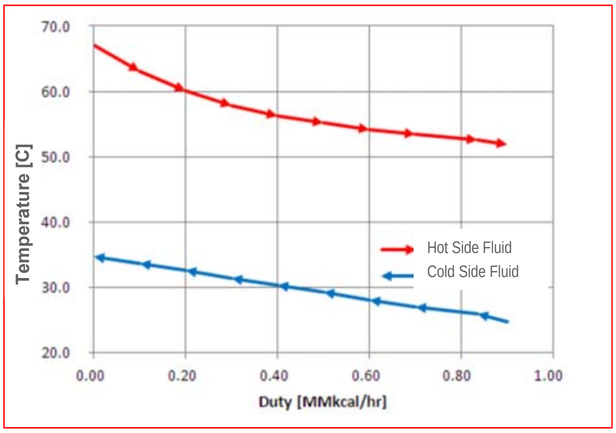
The LMTD Base is equivalent to a heat exchanger with a margin of almost 50%



Mixture Condensation (Saturate)

Consider an example in which a mixture in a saturated vapor state is cooled to a saturated liquid state by using CW of 25 → 35 [C].





[Study Conditions]
Heat Exchange Rate : 0.90 [MMkcal/hr]
Mixture : 67.0[C]→51.9[C]
CW : 25[C]→35[C]

LMTD vs WMTD (Saturate)

	Hot Side Fluid	Cold Side Fluid				
Heat Load	Temperature	Temperature	Zone ΔT1	Zone ΔT2	Zone LMTD	q / LMTD
[MMkcal/h]	[C]	[C]				
0.00	67.03	35.00				
0.10	63.13	33.89	32.03	29.24	30.62	0.0033
0.20	60.04	32.78	29.24	27.26	28.24	0.0035
0.30	57.86	31.67	27.26	26.20	26.72	0.0037
0.40	56.33	30.56	26.20	25.77	25.98	0.0038
0.50	55.17	29.44	25.77	25.73	25.75	0.0039
0.60	54.23	28.33	25.73	25.90	25.81	0.0039
0.70	53.41	27.22	25.90	26.19	26.04	0.0038
0.83	52.65	26.11	26.19	26.54	26.37	0.0051
0.90	51.91	25.00	26.54	26.91	26.73	0.0024
					Sum	0.0334

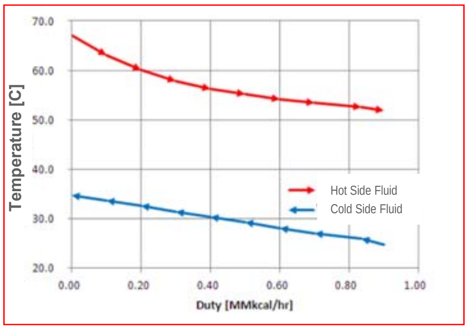
Overall		
ΔT1	ΔT2	LMTD
32.03	26.91	29.40

LMTD = 29.40 [m2]
WMTD = 26.83 [m2]

A LMTD = 61.05 [m2]
A WMTD= 66.90 [m2]

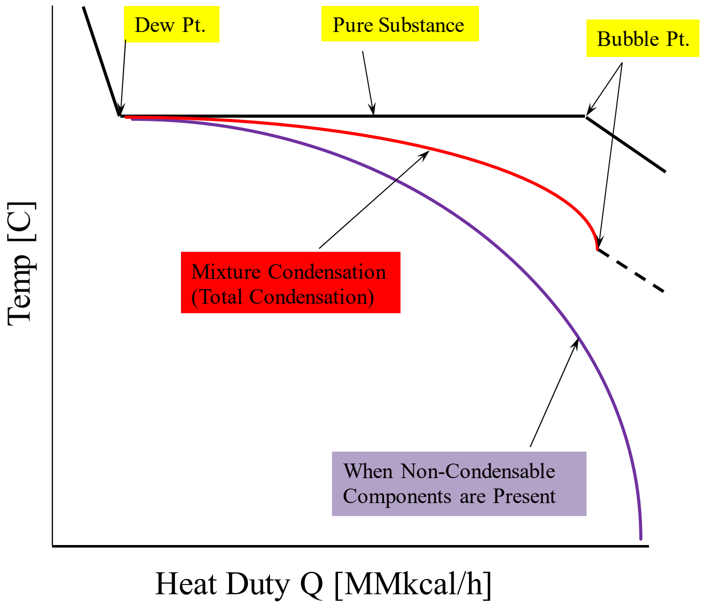
Ratio −8.7%

The LMTD Base is equivalent to about 10%, a heat exchanger with no margin



In cases where the temperature change is not linear, it is necessary to use the segmented weighted mean temperature difference.

Note: Temperature Change in Condenser



Under Constant
Pressure Conditions

In the hot fluid of a condenser, the dew point temperature drops due to the pressure drop. Also, in a condenser containing non-condensable components, the partial pressure drops due to the presence of non-condensable components, so the temperature drop can be considerable. It is necessary to pay sufficient attention to these points when studying condensers.

Conclusion

- The logarithmic mean temperature difference (LMTD) is derived from a state where the temperature difference vs the heat load changes linearly (single phase with constant specific heat).
- For this reason, the LMTD cannot be applied directly to a heat exchanger that has a phase change or where the temperature variation is not linear. For this type of heat exchanger, it is possible to improve the investigated accuracy by using the weighted mean temperature difference.
- However, the number of interval divisions should be changed according to the degree of the temperature change, and it is necessary to use a large number of divisions at places where the change is large.
- In heat exchangers with phase changes, the pressure greatly affects the temperature difference. Therefore, caution is required in addition to phase equilibrium calculations.